Lottery

In some lotto's, to win the highest gain, the winning ticket needs to have the same numbers as the numbers drawn by the ball machine. These numbers are a combination, a subset of the set of all whole numbers from 1 to 49. For a ticket to be winning, that the highest gain occur, all that's needed for that is that every number in the drawn combination is present in the subset that is the combination of the ticket. Said otherwise, both combinations must be the same, without considering the position of each number in the combinations (because the positions may differ). For each possible combination, we need to consider 6 different numbers, between 1 and 49. So, we can choose 1 number among 49 different numbers for the first number of the combination, 1 among 48 for the second, 1 among 47 for the third, etc, up to the last number of the combination, where we choose a number among 49 less the number of previous choices, giving 44 possible choices for the last number of the combination. Overall, we can choose a combination, among $49 \times 48 \times 47 \times 46 \times 45 \times 44$ possible choices, so, 10068347520 different choices. But this number contains many choices that are unneeded, because multiplications over whole numbers are commutative. Meaning, multiplications on whole numbers can be done in any order, they will always give the same result. The additional choices are different ways to multiply those numbers to get a given combination. They are how many different ways the numbers in a combination can be permuted. A permutation is a specific arrangement of the numbers of a combination. To get the real numbers of different possible combinations, we need to remove these extraneous choices from the number of possible choices. There's 6 different positions where the first choice can occur in a combination, and 5 for the second choice. 4 for the third, 3 for the fourth, etc. So there's $6 \times 5 \times 4 \times 3 \times 2 \times 1$ multiples of possible choices to choose a combination in the number 10068347520. Dividing 10068347520 by the factorial of 6, we get 13983816.

$$\frac{49 \times 48 \times 47 \times 46 \times 45 \times 44}{6!} = 13983816$$

In a draw, only one combination is chosen, among 13983816 choices. So, there's one chance over 13983816 that the combination of a ticket is the same as the chosen combination. If a ticket is valid for two draws, then the chances for the ticket to be winning for the first draw are the same. The chances for the second draw are the same as for the first. Because the ticket is now valid for two draws, there's now an additional chance that the ticket is now winning for both draws. If the ticket is winning only for one draw, to know the chances for this event to occur, we need to multiply the chances to win this one time, with the chances to lose in the other draw. Because it's possible to win one draw only at the first, or second draw, to know the total chances to win only one draw, we need to add together the chances to win the first draw only, and the chances to win the second draw only. To know the total chances to win at least one time, for those two draws, we need to account the chances to win both draws. Then, the total number of chances to win, if the ticket is valid for two draws, are:

$$\frac{2 \times 13963815}{13983816^2} + \frac{1}{13983816^2}$$

If the ticket is valid for three draws, it's a bit more complicated. The chances that the ticket wins for only one draw, is the chance the ticket is winning for one draw, times the chances to lose the other two draws. It's possible to win one time only, at only one of the three draws. It's also possible to win two times, at the first and second draws, the first and third draws, or the second and third draws. So there's three different possibilities to win two times. To know the total number of chances to win, if the ticket is valid for three draws, we need to add the chances to win for one, two, and three times. The chances are:

$$\frac{3 \times 13963815^2}{13983816^3} + \frac{3 \times 13963815}{13983816^3} + \frac{1}{13983816^3}$$

Now, we need to talk about Pascal's triangle, in honour of Blaise Pascal. But first, we will construct the triangle: We take one box, and we put the number 0 into it. We make an infinity of copies of this case, and put all of them on behind the other ones. We now have an infinite number of cases, each of them contains the number 0, in a straight line. After that, we take all these cases, and we make an infinite number of copies of them, and put any one of them under the other lines, shifting each copy half a case toward the right, so each case touch two case of the line over it. Now, for each case, we set the rule that it must contain the sum of the two numbers contained in the two cases over it that touch it. For a random case, we decide that the rule doesn't hold, and we put into it the number 1, so that it replace the previous number, 0. Then. Pascal's triangle has been built. Here's an example of the triangle, beginning with the case where we removed the rule:

$$\begin{array}{ccccccccc} &&&&1&&&& \\ &&&1&&1&&& \\ &&1&&2&&1&& \\ &1&&3&&3&&1& \\ 1&&4&&6&&4&&1 \end{array}$$

For a ticket that is valid for 4 draws, the process to know the chances to win is similar. We add the chances to win only one time, with the chances to win two time, with the chances to win three times, lastly with the chances to win all the draws. An array:

Number of winning draws	Number of ways to win
1	4
2	6
3	4
4	1

We remark in the array, that there's a relationship between Pascal's triangle, and the different ways to win any given draw, for a ticket valid for 4 draws. In the triangle, we are also seeing a relationship between the different ways to win for a ticket valid for 3 draws, and the fourth line of the triangle. In fact, we can also consider the third line of the triangle, to identify a relationship with the ways to win, for a ticket valid for two draws.

In the previous ratios, we can see that the numerator of each ratio is the chances to lose for one draw, times the number of draws, less the number of winning draws. We can then find a formula that will gives the numbers of total chances to win, for an arbitrary number of draws.

$$\sum_{i=1}^{n} \binom{n}{i} \frac{13963815^{(n-i)}}{13983816^{n}}$$

We replace the n by the number of draws the ticket is valid. We add successively the resulting ratios, replacing i with a number between 1 and n, and the sum is the total number of chances where it's possible to win the highest gain, for a ticket valid for n draws. The n and i inside the parenthesis beside the ratio is named the binomial. The n indicate the line in Pascal's triangle, and the i, the case. The formula may give the impression that the number of chances is big, but this number if in fact, very small. The total number of chances where the highest gain isn't won are:

$$\frac{13983815^{n}}{13983816^{n}}$$

If there's only one chance to win the highest gain, with a ticket valid for one draw, then there's all the other chances for not winning the highest gain. For each chance for not winning, there's all the chances possible for not winning at the other draws (if the ticket is valid for more than one draw). In all, if we buy a ticket valid for n draws, there's all the chances for the ticket to be winning, or not:

$$\frac{13983815^{n}}{13983816^{n}} + \sum_{i=1}^{n} \binom{n}{i} \frac{13963815^{(n-i)}}{13983816^{n}} = 1$$

So, gambling at the stock market may be more profitable.